Marvelous Mathematics How mathematicians wanted to improve the quality of life in Western Europe, 1945–1975

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Abstract. In this paper, we study the motives of the New Math reformers in Western Europe from the perspective of the ideas behind the moral commitment of mathematicians and their conviction that mathematics could improve the quality of life.

Báječná matematika. Jak matematici chtěli zlepšit kvalitu života v Západní Evropě, 1945–1975. New Math čili moderní matematické vzdělávání, byl reformní směr ve výuce matematiky na základních a středních školách v řadě zemí po celém světě. V tomto příspěvku studujeme motivy reformátorů v západní Evropě, a to z perspektivy myšlenek, které stojí za morální potřebou matematiků účastnit se reforem a jejich přesvědčení, že matematika zlepší kvalitu života.

Keywords: New Math ● history of math education ● teacher training ● 1945–1975

New Math, or modern mathematics education, as it was called in some European countries, was as varying in content as were its advocates. In many countries across the world, some form of New Math emerged during 1945–1975, either on paper or in practice, reforms taking place in both primary and secondary education. Invariably, the set theoretic language played an eye-catching, although not always fundamental, role in these reforms; new subjects being introduced into the curriculum, and a new approach towards the learning process were equally important. Following the 1978 volume of *Educational Studies in Mathematics*, where the New Math episode was reflected on for the first time, many papers have been devoted to the New Math episode, from a national perspective mainly focusing on either the mathematical contents [Bjarnadóttir 2016; Walmsley 2003; Noël 1993] or on the problems faced while implementing the new curriculum [Ausejo 2013; d'Enfert 2011; Noël 2002]. More recently, studies have been devoted to the particulars of specific key conferences [Schubring 2014; De Bock et Vanpaemel 2015] and to the specific motives of key players [Menghini 2015; Vanpaemel 2012].

A really novel approach is taken by Christopher Phillips [Phillips 2015]. He sheds new and intriguing light upon the New Math episode in the United States by analyzing the rhetorics used by the various players, thereby noting that Cold War America was a battlefield. The New Math war was waged with words, rather than with the threat of weapons, but nevertheless: a war it was. The war waged between left and right wing idealists, whether they were mathematicians, educational scientists, politicians or teachers. The battlefield was the classroom.

Phillips depicts the rise and fall of New Math as a political story, against the background of rising mass communication and a, to parents, unsettling new youth culture. In doing so, he also makes the (equally political) ensuing "Back to Basics" movement almost seem self-evident. According to Phillips the US anti New Math atmosphere was the result of political and social unrest. Although heralded as the solution to all the problems the US were facing in the 1960s, parents only a decade later, regarded New Math as an overly academic exercise, which would not bring the nation the social coherence that was so needed. Math itself did not fail: instead the academics that had fallen from grace, it was the new educational attempt that was trusted no longer. "Back to basics" would help set things right, offering, in a way, a return to tradition, illustrating a decline of popular faith in federal government and science as such.

Obviously, there was a political aspect to New Math in Western Europe as well, if only because pedagogues and psychologists were getting a foothold in teacher training, to the detriment of mathematical training. But much more than some minor squibles and the deliberate (?) denial of one psychological current in favor of another [Schubring 2014; De Bock & Vanpaemel 2015; Vanpaemel 2012, p. 11] can hardly be explained by that: on several occasions, psychologists and mathematicians were striving for the same course: there was coherence in the conunundrum of opinions and attempts by New Math protagonists. Moreover, faith in science and government was what kept Europe together. As was noted by Vanpaemel, it is necessary to understand the ideas behind the moral commitment of mathematicians, their ideas regarding the way(s) that mathematics would actually improve the human condition, to get to grips with the motives of the reformers [Vanpaemel 2012, p. 8]. It is this aspect that is dealt with in this paper.

One of the things that is noteworthy in the New Math episode is the concern with educational matters from the side of university mathematicians. Of course there had been interest in educational matters before, for example from the education committee of the *Union Mathématique Internationale*, but this, at least in most Western European countries, had always been descriptive – interested, more than intervening, helping instead of steering. In the decades after 1945, there was a tendency by several Western European mathematicians to claim expertise in the field of secondary (and sometimes even primary) math education. Some of them actually started intervening [cf. De Bock et Vanpaemel 2015, p. 34]. Using literature about the French, Belgian and Dutch situations, this paper focusses on the motives of mathematicians to get involved in the New Math movement on the secondary school level. Why did they participate and what did they aspire to achieve? What were the driving forces that made them want to intervene?

All the key players were well aware of the fact that they were living in a time that they could actually make a difference. When in 1965 Hans Freudenthal asked his Begium and French (and many other) colleagues to join the editorial board of his new journal (*Educational Studies in Mathematics*), many of them joined enthusiastically, showing in their replies that they were well aware of the fact that renewal was taking place [Freudenthal Archive, inv.nr. 1785]. Academic mathematicians actively participated in curriculum discussions and consciously took it upon themselves to help restructure math education.

This paper will start by sketching the mathematical scene of post war Europe, to continue on the educational systems in the various countries, focusing on the role of mathematicians (but not on the role of mathematics in education). Afterwards, four topics that were reflected upon by all the mathematicians participating in the educational reforms, will be addressed. The topics addressed reflect also the concerns participating mathematicians had, and will therefor shed light upon their motives. Finally, in two paragraphs, some concluding remarks will be made. First on the ideals that drove mathematicians to participating in the reforms, and second on what made most of them leave.

Mathematics in the twentieth century

Mathematics had become one of the driving forces of modernity, or at least that was the way that many intellectuals in the 1940s and 50s had come to look at it. During the early days of the twentieth century one of the academic manifestations of mathematics had taken a new approach. Set theory had become its major language, structure its main topic, proof theory well thought-through. Although certainly not the first text in this vein, one may regard the two-volume *Moderne Algebra* (1930–1931) by Bart van der Waerden as the pivotal text. The algebra by Van der Waerden was no longer about solving equations, it was about rings and fields. Algebra was no longer a generalization of arithmetic, although arithmetic could still be regarded as a special case of an algebraic structure, it was the structure itself, and the way objects related to one another within this structure, that had become the object of mathematics. Structure had become the subject matter of mathematics, the similarities or differences of structures the new focal point.

Jean Dieudonné, André Weil and other interwar French mathematicians joined in a group, that took this new view of mathematics as the point of departure for a completely new and revised mathematical framework: building mathematics from scratch. This group published und the pseudonym Nicolas Bourbaki. The Bourbaki manuscripts circulated before and during the war, and were very well thought-through by the time they were published, from the 1950s onwards. In these publications, the language and foundation for mathematics was set theory, the subject of mathematics was structure. Although not every mathematician regarded this the essence of math, all did marvel the beauty of the attempt, did appreciate the unifying spirit of the project. Bourbaki stood for a new idea of what mathematics was about [Phillips 2015, pp. 50–51]. In France, Belgium and the Netherlands even more so than in the United States [De Bock & Vanpaemel 2015, pp. 166–167].

At the very same time, mathematics became more and more an applied science. This was visible in the work of mathematicians like Jan Tinbergen, who created economical models in the 1930s, and David van Dantzig, who did work in statistical analysis in the late 1940s and 50s. Mathematical applications were even more widely received and appreciated in descriptive statistics and computing [Alberts 1998; Alberts 2000].

The mathematical conscience in the post war period was bourbakist in essence, to quote the French educational reformer André Revuz [Revuz 1996], or was it? Indeed, many of the mathematicians involved in the reforms were topologists or logicians. But the reform movements in the three countries dealt with here, varied in the way they adhered to Bourbaki ideals. Leaving aside individual preferences and changes of opinion, in France the educational reform movement was most outspoken favoring Bourbaki. Here, the reform was top down, guided by academic mathematicians [d'Enfert 2011]. In Belgium mathematicians were accompanied by mathematics teachers who had received academic mathematical training and taught in the upper classes of secondary schools. The bourbakist mentality was both reinforced and tamed by school teachers' pragmatism [Vanpaemel 2012]. In the Netherlands, reform came from various sources, and it took several committees considerable effort to get people to cooperate at all. Academic mathematicians realized that any curriculum or way of teaching, wouldn't become a nation-wide standard – not without considerable effort. Bourbaki exactness was not considered a suitable way of expression per se by all Dutch mathematicians, since it was the result of many years of mathematical development. It was considered suitable, though, for those pupils who aspired to attend university education [Beckers 2016, pp. 134–136]. This was, of course, the group of secondary school pupils where mathematicians could most easily claim some expertise.

To professional mathematicians, the unity of mathematics was best illustrated by the work of Bourbaki. Although not every mathematician would have recognized his work in these volumes, they could relate to it, at least metaphorically, as to a foundation of their work. The temple of mathematics was a widely used metaphor in France [Le Lionnais 1948]; the tree of mathematics became a convenient and attractive metaphor in the late 1950s, to those who thought the temple was too secretive or exclusive [Phillips 2014]. In both cases, it was clear that mathematics had its roots in society, was to serve society and played a decisive role in society, and therefore deserved all interest from educators. This view of mathematics, as a science playing a vital role in both social and technical developments, whether it was considered to be a pure science or an essentially applied form of knowledge, was widely held among intellectuals [Alberts 1998, pp. 134–138].

Mathematics education

Early Twentieth Century school systems in western Europe were aimed at social segregation. Elite, civic and lower education were kept apart, which was inherent to society being organized in segregated social strata. This is relevant, since, not only did it yield various mathematical curricula, reflecting this social stratification, it was also exactly this feature that started shifting. Especially after WW II, the changing socio-political view, favored a unified school system in all Western European countries, France, Belgium and the Netherlands in particular. Unifying, not only in the sense that pupils from all socio-economic backgrounds were expected to have, at least theoretically, equal opportunities within this school system, but also in the sense that they should have access to the same, mixed (!) classrooms. Governments in the three countries recognized, or were convinced, that mathematics was to play a crucial role within western culture, and therefore were always willing to pay for or listen to plans or initiatives concerning math education. Sponsored by UNESCO, OEEC and national governments, some of these initiatives blossomed.

In Belgium, shortly after the war, there was an initiative taken by the school of Ovide Decroly. There the *Comité d'Initiatives pour la Rénovation de l'Enseignement* was founded in 1945, by teachers from the École Decroly and the *Université Libre de Bruxelles*. Among them was the geometer, communist and educational reformer Paul Libois (1901–1990) [Schandevyl 1999]. He and Willy Servais (1913–1979) took the initiative of taking the educational reform ideas to the level of the Belgian government [Vanpaemel 2012].

Discontent with the math curriculum was present in France as well. Shortly after the dust of the war had settled, the French mathematician Caleb Gattegno

(1911–1988) took the initiative of founding the *Commission Internationale pour l'Étude et l'Amélioration de l'Enseignement des Mathématiques* (CIEAEM). Members were people from mathematical academia, like the French topologist André Révuz (1914–2008); they were soon joined by other European academics (and math teachers with a university background), such as the Belgium group theorist George Papy (1920–2011) and the Dutch logician Evert Willem Beth (1908–1964). From April 1950 onwards, they gathered once or twice annually, to discuss possibilities for math reforms, and quickly grew out to a European group of academics, concerned with mathematics education [Felix 1985]. Their conferences resulted in actual textbooks. Papy's *Mathématique Moderne* was inspired on his attendance of the CIEAEM gatherings [Papy 1964], and this book, in turn, inspired many textbooks in Belgium. His slogan, "Les mathématiques du Papy ou les mathématiques de papa!", illustrated the rhetoric that accompanied curriculum reform. CIEAEM was an academic group that had crystallized around a common topic. From the side of the French government, the subject of math education became important later. In 1967, they installed a commission, presided by the geometer André Lichnérowicz (1915–1998), with the purpose of advising on the math curriculum reform. A more permanent *Institut de Recherche sur l'Enseignement Mathématiques* (IREM) was officially founded in 1969.

In the late 1950s Belgium and France revised their school systems, adapting it to modern society. Reforms of curriculum and teaching practice came later [Tyssens 1999; d'Enfert 2011]. The Netherlands followed with new legislation in 1961, effective from 1968 onwards, changing the structure of both primary and secondary education profoundly, after some minor changes to the curriculum in the early 1950s [Smid 2015]. In Belgium and France, therefor, curriculum reforms had to give rise to a change in the character of secondary education, whereas in the Netherlands these changes were generally presented as part of a "package deal": the logical consequence of a school system being put upside down.

In the Netherlands the New Math movement was more bottom-up. Some academics were involved out of personal interest, most noteworthy the geometer N.H. Kuiper (1920–1994) and the aforementioned E.W. Beth. In the 1960s they would be joined by the topologist Hans Freudenthal (1909–1990). Dutch academics were much more modest in their approach, compared to their Belgian and French colleagues. For example, Kuiper, in the early 1960s stated that, according to him, Dieudonné's "Down with Euclid" (the catchy battle-cry that the well-known bourbakist had used in support of his vision of New Math) was the most interesting new perspective, explicitly welcoming the participation of Dutch math teachers in an international project for curriculum reform [Kuiper 1961, p. 265, 276]. So he left the implications for both curriculum and teaching to the math teachers and did not claim expertise – although he was very well informed. It was only in 1961 that a government committee of academics, mostly mathematicians, was formed to look into the possibilities of reforming math education, the *Commissie Modernisering Leerplan Wiskunde* (Commission for Modernization of the Math Curriculum, CMLW). From that moment on, also Dutch academics openly intervened in matters of math education [Wijdeveld 2003; Beckers 2016].

Math curricula in Europe were subject of reflection and debate already before the war. After the war the debates intensified, mostly because it was becoming evident that actual reforms were going to take place. Reflection on what was considered to be essential to math education therefor became a common theme.

Reflection thinking

Since the rise of pure math, thinking had been the mathematicians' business. Certainly before the war, it was common to work from the assumption that learning mathematics would stimulate sound reasoning. It was not very common among mathematicians to reflect on the nature of thinking otherwise than in mathematical terms. That mathematicians in the 1950s started doing so, indicates that they became aware of, and were genuinely interested in, educational problems.

There were various ways in which reflection on what the actual thinking act was, took place, the most common being the psychological and the cybernetic approach. Both made use of research from the 1930s, that had shown time and again, how pupils who had learned arithmetic or algebra, were not capable of using that knowledge to their advantage in simple (new) situations, or made very elementary mistakes in applying arithmetic or algebra to a real life situation. A popular story that went around in several versions was about a "real life" situation in the post office, where the man behind the counter had to figure out how much the customer had to pay for 23 stamps of 17 cents each. After having multiplied he received a banknote and the man starts counting how much he has to return to the customer. When he finally has figured it out, the customer decides that 22 stamps will do as well. The man behind the counter consequently started redoing his work, starting from 22 times 17 cents, instead of simply returning an extra 17 cents [for example: Freudenthal 1963, p. 36]. Apparently, being good at arithmetic or solving equations, did not prepare pupils better for real life. Learning algebra the "old way, hadn't helped this pupil to think in this situation.

What was thinking? The CIEAEM had started from the conviction that the internal logic of mathematics would force pupils to start thinking [Félix 1985]. Psychologists, educational scientists most notably, opted for a definition entailing a broad spectrum of habits towards life and knowledge. Reading and writing, expressing one's opinion in a certain way, both orally as on paper, where part of that. But also the ability to acquire new knowledge, being able to apply knowledge – most notably in "new" situations – and reflecting on an answer obtained were important. The theories of Jean Piaget (1896–1980), Pjotr Gal'perin (1902–1988) and several other psychologists, tried to pinpoint what the actual thinking process was, and how it could be stimulated, how it could be learned. They went beyond the traditional view, which had been widely accepted before the war, that regarded thinking as copying or remembering (word) associations.

Thinking, in whatever form intended, was essential to the new education. How did pupils learn to think? That was not, altogether, an easy question to answer. Educators were convinced that intelligence was not evenly distributed, but most of the continental European academic elite, the mathematicians being no exception, were convinced that in the traditional school system, talent was wasted. Among the working classes there was talent, but it couldn't blossom, since so much cultural luggage was expected from pupils in the socially upper forms of education. Mathematics, contrasting to the humanities, was ideally suited to overcome that disadvantage [Armatte 1996].

If there was a reason for making mathematics the core of the new curriculum, mathematicians were aware of the fact that they had to show how mathematics would overcome transfer problems. In other words: how would a revised mathematics education succeed where it had failed in teaching pupils even elementary arithmetical thinking before? In more modern conceptions, new ideas about thinking were also part of CIEAEM conferences in April 1950, were the program was "relations entre les programmes de mathématiques des écoles secondaires et le développement des capacités intellectuelles de l'adolescent", and more explicitely in April 1952 ("Structures mathématiques et structures mentales") and July 1953 ("les rapports entre la pensée des élèves et l'enseignement des mathématiques") [Félix 1985].

One thing was clear, and all participants in the discussions agreed upon this: learning to solve equations was not the same as learning to think!

Reflection on mathematical thinking

Some virtuosity in solving equations, however, was considered essential to mathematics. What was the essence of mathematical thinking, that made it an essential part of western education? The answers to that question were diverse, as were the people that formulated answers. The classic text by Stanford mathematician George Polya (1887–1985) and its many revised reprints [Polya 1945], describing mathematical heuristics, was a guide to many mathematicians.

But important as solving problems was, it was not equivalent to mathematical thinking. The 1950s and 1960s witnessed several reflections on mathematical thinking in relation to thinking. All had left the naïve idea that mathematics would induce sound reasoning. At the very least, it took some extra effort by the educator, to impose the mathematical sound ideas upon problems in the real world. Roughly, one might discern three groups of ideas.

First, there were those, who did not believe that learning mathematics would teach pupils to think at all. Learning to think mathematically, however, was an essential goal of mathematics education, according to most of the educators involved, even to those who did not believe it would induce thinking. "Mathematical thinking", to this group of mathematicians, was close to applying and evaluating heuristic strategies [Skemp 1971]. The important role of mathematics in western society to these mathematicians was enough reason to grant math its central position in curricula. It were mathematicians like Caleb Gattegno who voiced this opinion, even after the New Math frenzy had died and they looked back, somewhat disappointed, with the result:

> Like Miss Félix, I would like to say that the real motivation for all those people who got involved [in CIEAEM], including myself, was the feeling that no-one should be deprived of the joy of discovering mathematics. We were convinced that this would be accessible to everyone, because we had experienced this joy ourselves. [Félix 1985, preface; Búrigo 2015, p. 101; translation: DB. Original text: Comme Mademoiselle Félix, je puis dire que le vrai motif derrière cet engagement de tant des gens, y compris moi-même, est le sentiment que personne ne devrait être privé de la joie de la découverte mathématique que nous savons être à la portée de tous parce qu'elle a été à notre portée].

Second, there were those, who believed that learning the possibility and necessity (or superiority) of a mathematical approach, learning pupils to "see" the mathematics in the world around them, was the essence of mathematical thinking. Instead of focussing on the mathematics itself, educators also had to pay attention to mathematizing real life problems. This could either take an old fashioned "applied mathematics" form, or a more modern approach, using guided invention principles. New subjects, such as statistics, number systems, linear programming and graph theory, became popular subjects. These were subjects any mathematician could relate to. Whether, and in what way, they should be part of secondary school curricula, was what was point of discussion.

Third and finally, there were those, who were convinced that the example of mathematical reasoning, if taken from the most elementary mathematical structures, would convey a certain and very revealing introduction to the laws of drawing conclusions. Starting from simple structures, such as groups, or even more elementary structures, the pupil was to be introduced to the laws of algebra or arithmetic. Mathematical, or logical, simplicity, was taken as a didactically suitable approach, because it burdened the memory less; but it also acquainted pupils in an early stage to be cautious in applying thinking steps or theorems blindly. Looking for mathematical simplicity, for example, yielded results such as the linear algebra approach to statistics by Marc Barbut (1928–2011) [Barbut 1967], instead of taking measure theory as a starting point [Armatte 2012].

One way to set mathematical thinking apart from other forms of thinking was by the language in which the thinking was expressed. By using the language and symbols of set theory as the basis of all mathematical reasoning, the subject itself was envisaged to distinguish itself positively from all other, less precise, forms of reasoning. Using a specific "math language", would remind pupils that the subject was exact – and only logic was allowed. French academics, for example, advised using different words for concepts in the real world and the equivalent object in a mathematical model. This would help pupils both to keep the mathematization process (or reality) in mind, and at the same time realise what logical steps were allowed: no recourse to belief or suggestion was allowed [Gispert & Schubring 2011, p. 96]. This approach worked out good in the 1950s and 60s, when reform experiments took place in the higher classes of the lycees.

Belgian mathematicians opted for almost the same approach. The colorfully illustrated books by Papy were an example by which the Belgian mathematicians could convince their teachers. These teachers were mainly those from the gymnasia, not the (non university trained!) teachers from vocational training colleges, since they were not involved in the New Math courses [Vanpaemel 2012]. Dutch mathematicians in the 1960s and 70s admired the work of their French and Belgian colleagues, but thought the set theoretic language a nice option, which should only be introduced to those pupils who were actually in need of it – i.e., those destined to go to university. An entirely new language, according to them, would burden the mind of many pupils too much. Using clear and distinct phrases from their mothers' tongue, however, was valued by several Dutch mathematicians (as well as math education reformers) [Beckers 2016].

Traditionally, geometry had been the subject where mathematical thinking was at its best. It was, therefore, in geometry that the New Math approach, would become most visible and most disputed. In France the Lichnérowicz commission, entrusted with designing a new program, had been rather successful in its work for the higher echelons of secondary education – where the new curriculum and teaching methods were readily accepted by university trained mathematics teachers. In the lower grades, however, most teachers had no knowledge of modern math. Trying to impose a revised geometry program for the lower grades, starting from the mathematically simpler form of affine geometry, the commission met its Waterloo in the early 1970s: the head of the commission, and finally the entire commission, resigned, and new plans were made [Gispert & Schubring 2011, pp. 97–98]. In Belgium, however, the new program met with less opposition [Vanpaemel 2012].

Euclidean geometrical reasoning had been at the core of the mathematical thinking for centuries. Introductory courses in geometry, using materials to illustrate certain aspects of geometrical objects, were very popular in the 1950s. It made pupils obtain some intuitive experience towards the objects, that came under more abstract consideration during geometry classes. These kind of ideas were not new: they had been around also before 1939, but it had never served as the basis for an explicit attempt to teach mathematical thinking – that was considered to develop naturally with every proof the pupil understood. It was exactly with the goal of recognizing when pupils were thinking mathematically that, for example, Pierre van Hiele (1909–2010) wrote his dissertation. Mathematical thinking, to him, was equivalent to mathematical understanding. And this was, he explained, best observed when pupils were at work, by analyzing the words they used, and the meaning(s) they attached to these words. Mathematical understanding was independent of the language thoughts were expressed in, but higher levels of understanding did require more sophisticated expressive possibilities. This could be stimulated (not substituted!) by having pupils experiment with objects [Van Hiele 1957].

Although playing with geometrical objects was valued as an introduction, to French mathematicians, the set theoretic language itself was the key to mathematical thinking. They even argued that Latin as a formal language could be replaced by the "Esperanto of mathematics", as the language of set theory was dubbed, for the pupils at the lyceum. Acting such, the pupils from the higher classes would be just as well equipped rhetorically as they used to be, and there would be a more democratic (or meritocratic) access to the higher classes of secondary education [Armatte 1996]. This touched upon another point mathematicians reflected upon: the educational system.

Reflection on education

There were at least two levels on which reflection on education took place among mathematicians. There was reflection on the desired goals of education, but mathematicians reflected also on the way these goals were best achieved and how achievements could (or could not) be measured.

Goals of education changed all over the post war continent. "Education for all" was the key. This did not necessarily mean that everybody was offered the same education, but creating more equal chances was definitely in the minds of most mathematicians involved in these discussions. It was a more widely held view, originating from educational researchers, who were afraid that educational systems were not fitted to meet the needs of modern society. They observed that modern society was becoming increasingly more complex, and it required for example, that people would continue to learn, also after formal education. In a sense, they had to learn how to master their own educational process. Furthermore, changes in society, required people to become more flexible workers: low cost simple labor was going to disappear, since it was being taken over by automation. This meant that labor forces were still needed, but on a higher level. The fact that educational systems were not at all prepared for that, made intellectuals fear for a world educational crisis [Coombs 1968]. Although somewhat put in perspective in the 1980s, it was continually stressed that educational reform was needed [Coombs 1985].

Mathematicians generally thought their subject was ideally suited to be inclusive towards pupils from all social backgrounds, because it assumed relatively little prior knowledge. In France "mathematics education for all" meant for all who could actually do mathematics well, and some form of math for the rest; in the Netherlands mathematicians tried to address as large a group of pupils as feasible. Belgium was somewhere in between. In all countries, "education for all" stimulated reflection on which subjects to teach, in order to be relevant for all future professions. Of course math, or rather, mastering certain mathematical techniques, was relevant to technology. Individual learning projects and group projects, using electrical circuits, mirrors, or other equipment, would show the relevancy of math to those interested in future technical professions rather easily. To other pupils, working with the materials could stimulate them to do math [Gispert 2003; Nabonnand 2003]. But mathematics also had to contribute to a more exalted way of learning to think. Some mathematicians liked the idea that mathematics also contributed to democratic values. This in the sense, that to the mathematician, honesty and truth were self evident, and authority only counted on the basis of proof [Gispert 2011; Vanpaemel 2012; Beckers 2016].

This being said about the goals of education, these new goals, so it was taken for granted, needed new ways of teaching. Discovery learning, where the teacher was required to follow the thoughts of his or her pupils, by careful posed questions, guiding them to a solution they could call their own, was an idea that had been introduced into mathematics by Beberman [Walmsley 2003, pp. 34–35]. Programmed instruction was taken seriously in the US [Zoll 1969], certainly since IBM started investing in it in the 1960s [Young 1968; Glaymann 1968; Thwaites 1970; Buck 1995]. Europeans looked into the possibilities of programmed instructions – although generally without the use of machines, but with specially designed books, that allowed pupils to skip parts, or do extra exercises if certain steps turned out to be difficult. Films, television and radio broadcasts, were widely used in teaching math by Europeans. Of course, these new ways of teaching helped teachers to cope with the different levels of insight into mathematics within their classroom. By having children more at work for themselves, teachers had time to spend on those who needed extra attention, or on those who needed more math. To the mathematicians that are of concern here, all these novel teaching methods somehow had to contribute to the main question: How to teach mathematics, so as to be useful?

This question was at the heart of a conference that was organized by Freudenthal in Utrecht in 1968. He himself posed the question, but did no attempt to answer it. He did, however, suggest that to most pupils mathematizing (a part of) reality would be a great accomplishment; mathematizing mathematics itself, i.e. axiomatisation, was neither feasible, nor desirable as a goal for every pupil [Freudenthal 1968]. At the same conference, his Belgian colleague Servais, however, did answer the question, and in doing so even addressed both utilities of mathematics: that of solving "real life" societal problems, and that of serving as a perfect example of reasoning. According to him, both were best served by offering pupils activities that would require them to make distinctions between the mathematics and what it was that was being mathematized. The ideas of mathematizing (leaving out non essential characteristics, focusing on relevant and measurable issues, thinking about the relationship between these etcetera) were just as important as axiomatizing, where the pupil would be ordering and systematizing his results [Servais 1968]. Of course, Servais only had Freudenthal's gifted pupils in mind. To all participants in the conference it was clear that new ways of teaching had to be explored and new goals had to be set for the math curriculum.

Reflection on the role of the teacher

These new goals should be manifest in new teacher training. Both the new mathematics, based on set theory and structures, and the new applications of mathematics, such as more sophisticated mathematical techniques, discrete and numerical mathematics, were new to most teachers.

New teacher training was one problem. Whereas before the war, mathematicians found a job in secondary education, this was no longer evidently the case in the 1950s. The new role of mathematics in society implied that many trained mathematicians found their ways into industry – a fact that was already noted in the 1950s [Freudenthal 1956, p. 238].

It was quite clear that teachers were the key to every new curriculum. Mathematicians thought it worthwhile to make teachers aware of the direction that present-day math research was taking, or they had to be made susceptible to the subtleties and power of formal set theory. If nothing else, that would at least make the gap between academic and secondary school mathematics shrink. Ideally, however, it would also help improve the didactics. In all cases it was necessary to have teachers trained specifically in the modern day math. French mathematicians started additional teacher courses from the sole perspective of making them acquainted with present day research in the late 1950s, but soon new didactical perspectives were added to the program [Revuz 1969].

Educational programs, heavily funded by UNESCO and the OEEC, rose in the 1960s. The annual Belgium teacher conferences, introducing the new mathematics to teachers, were a great example. Many academic mathematicians played their part in these conferences, most of them from France, Belgium and The Netherlands. The well-trained teachers of the upper parts of secondary schools, regarded these courses as stimulating. The course syllabi were translated in various European languages, and distributed almost for free to everybody who wanted to receive a copy [Wiskunde 1960–1962]. The Dutch government supported various programs for extra teacher training, as initiated by the CMLW. Although rather successful as well, the government was soon overwhelmed by the amount of resources that were requested by the mathematicians to continue these trainings [Beckers 2016, pp. 130–131].

Mathematicians were generally inclined to see the role of the teacher as a potentially positive one. It was the teacher who could get the pupil's thinking on the right track, who could help to spot the weak points in his reasoning by asking the right questions, and who could actually judge the mathematical capabilities. Reinforcing the role of the mathematics teacher by teaching him more mathematics, making him or her aware of what modern mathematics was about, therefore became a logical consequence of New Math.

On the other hand, psychometrics tried to eliminate the subjective role of the teacher in assessing the qualities of the pupil. They had noted the ways that educational systems worked to keep the social status quo intact [Busato 2014, pp. 262–263]. Mathematicians agreed only so far as that this did not concern their subject, by simply stating that anyone who would cooperate in such practices, could never be a good mathematician, nor a good math teacher. The perfect math teacher would not be biased by social status: only mathematical proof counted.

How math should have improved quality of life

The most remarkable, perhaps, was that mathematicians did get involved in these discussions. All of them were genuinely convinced that they would contribute to the quality of life of the people in their respective countries. Where before the war, mathematician's concern with society (if any) were mainly on the level of applied mathematics, after 1945, math education was increasingly becoming a domain where one could show one's concern.

Mathematicians joined the post war efforts to improve mathematics education because they were offered the opportunity: popular belief in their science was at its peak. Helping in rebuilding the nation was an obvious academic task and education seemed to be the place to do it. Governments asked them to take part in committees (France and the Netherlands) or accepted their help gladly (Belgium). Although from various political backgrounds, all their contributions originated from a genuine belief, if not a conviction, that education, mathematics education in particular, had to help improve western values so as to prevent the dictatorial excesses recent history had witnessed. It also had to prevent the average citizen from "dropping out" of the educational system. Math classes would offer equal opportunities and would help mankind adapting to modern society, in the sense, that math was what made the world go round – either in naïve or in a more exalted way. Mathematics, in one way or another, was to improve the quality of life.

In what ways did mathematicians think math education should have improved the quality of life? Some mathematicians were convinced that an entirely new way of teaching was necessary, for mathematics actually to become the perfect breeding ground of modern society. Others thought that some new subjects, dressed up in the language of set theory, bringing the subject closer to the academic discipline, would do the job. Some thought both. Introducing new subjects in the curriculum, such as statistics, linear programming and computing, were obvious novelties, that were intended to prepare the future generation to recognize the marvels of mathematics. It would help citizens to cope with the demands to which they were confronted by modern society.

Introducing mathematics in new ways had an obvious advantage as well. Whether by a textbook that left more work to the pupil, offered a more playful introduction, went to a (much) more abstract level by introducing groups and rings, or a combination of these outlooks, what all authors tried to achieve was a more meaningful introduction to the subject. More meaningful in the sense that more pupils were addressed, better understanding (whatever was meant by that) was reached, or better (less faulty) application of math to real life problems was achieved.

Could math have improved the quality of life?

Several mathematicians look back at the New Math episode with regret, either because they feel something slipped through their fingers [cf. Félix 1985], or because they feel they've been wronged somehow [cf. Revuz 1996]. Others, like Papy, succeeded in clinging to their own truth. Some, like the Dutch mathematician Kuiper, simply left the math education scene, because they didn't feel comfortable with the politics [Takens 1995, p. 57]. Others, like Freudenthal, moved along with the changing tide [Beckers 2016, pp. 138–140]. And finally, there were those, like Servais, who had always seen the curriculum (and way of teaching) as something evolving over time [Vanpaemel 2012, p. 11]. Mathematicians, at least in the countries discussed here, by the end of the 1970s, had no obvious role to play anymore in mathematics education.

Politics and education are not mathematics. That was the harsh lesson learned by some of the enthusiastic reformers of the 1950s and 60s. It was the American mathematician Alan Bishop (*1937), who in 1990, in a paper reviewing two publications on math education, noted that mathematical science and mathematics education did not always "see eye-to-eye" [Bishop 1990, p. 151]. The rise of the new psychological disciplines of educational studies and psychometrics was one of the key factors in understanding the change in climate. In fact, there were incompatible paradigms between the mathematicians and the emerging field of educational science in the 1970s. Educational scientists wondered how to eliminate the subjective role of the teacher in the way that pupils were assessed, and for this reason developed statistical techniques, focusing on multiple choice questions, making visible ever more minute details in learning processes. Contrastingly, mathematicians were focusing on the teacher himself, many of them completely unconvinced of any social bias, their subject of study, after all, being completely free of unclear presumptions.

Those marveling mathematical beauty slowed down the process of reform by meticulously trying every possible solution and examining the results. Although inherent to the work of a mathematician, the strive for absolute truth did not blend well with the real needs from politicians. In Belgium, where mathematicians had a rather strong foothold in curriculum decisions owing to their close cooperation with teachers, this worked best. Since the Belgian government had not invested as much in math reform, they stood by when educational scientists in the 1970s took over the agenda from the math reformers. In the Netherlands, however, government officials openly expressed their annoyance over the slow progress of math reforms. Faced with growing budgets, the Dutch government started favoring another road, placing the mathematicians' ideals on a side track, actively stimulating a new pedagogical turn in education. In France, mathematicians kept some foothold in education within an expanding IREM, but were faced with a similar change in focus. Mathematics education, however marvelous its subject was, by the end of the 1970s was no longer a mathematicians' business.

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Summary

New Math, or modern mathematics education, attempted to reform mathematics teaching in primary and secondary schools in many countries across the world. In this paper, we study the motives of the reformers in Western Europe from the perspective of the ideas behind the moral commitment of mathematicians to participate in the movement and their conviction that mathematics could improve the quality of life. The main topics that were reflected upon by all the mathematicians participating in the reforms include mathematical thinking, goals of education and how to achieve and measure them, and new teacher training. In concluding remarks, we discuss why later most mathematicians abandoned the reforms.

Resumé

New Math čili moderní matematické vzdělávání, byl reformní směr ve výuce matematiky na základních a středních školách v řadě zemí po celém světě. V tomto příspěvku studujeme motivy reformátorů v západní Evropě, a to z perspektivy myšlenek, které stojí za morální potřebou matematiků účastnit se reforem a za jejich přesvědčením, že matematika zlepší kvalitu života. Mezi hlavní diskusní témata reformních matematiků patřily aspekty matematického myšlení, cíle vzdělávání, zejména jak jich dosáhnout a jak jejich dosažení měřit, a také reforma vzdělávání učitelů matematiky. V závěru diskutujeme, proč většina matematiků opustila reformní hnutí.

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